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An $\mathcal{N}=1$ superfield action for M2 branes

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ABSTRACT

We present an octonionic $\mathcal{N}=1$ superfield action that reproduces in components the action of Bagger and Lambert for M2 branes. By giving an expectation value to one of the scalars we obtain the maximally supersymmetric superfield action for D2 branes.

1. Introduction

Despite its importance for M-theory and higher-spin gauge theory holography[1] $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence is essentially unexplored in comparison to its $\mathrm{AdS}_5/\mathrm{CFT}_4$ counterpart. Presumably, the main reason has been the lack so far of a manifestly maximally superconformal invariant $(2+1)$-dimensional theory similar in status to $\mathcal{N}=4$ SYM in $3+1$ dimensions. It seems possible that this obstacle has been overcome by the proposal of Bagger and Lambert [2–5] of a $\mathcal{N}=8$ superconformal theory in $2+1$ dimensions, with a non-standard gauge structure based on 3-algebras. This theory has attracted very much interest in the past few months and many interesting results have appeared. Multiple M2 branes have been discussed in [7–11,13,14,16,17,28,30,41]. The relation of M2 to D2 branes was elucidated in [7,12,23,25,26,43]. Algebraic aspects of 3-algebras have been discussed in [6,15,19,22,24,33,37,38,40]. General aspects of three-dimensional Chern–Simons theories and extensions have been discussed in [20,36] and also in [18,21,27,29,31,32,34,35]. Very recently, an interesting class of $U(N)\times U(N)$ Chern–Simons theories that may describe multiple M2 branes have been discussed in [39,41,42].

In this short note, we present an $\mathcal{N}=1$ superfield action whose component expansion gives the BL theory for a 3-algebra with totally antisymmetric structure constants f^{abcd}. We use real three-dimensional superfields both for the matter as well as for the Chern–Simons part of the action. The crucial point is the use of the octonionic self-dual tensor in the construction of the real superpotential. In this way, the superpotential is only manifestly $SO(7)$ invariant. However, for specially chosen couplings, the component action coincides with the BL action, and hence full $SO(8)$ symmetry is restored. We believe that octonions will play a fundamental role in future studies of $\mathrm{AdS}_4/\mathrm{CFT}_3$.

Our motivation comes in part from corresponding studies in $\mathrm{AdS}_5/\mathrm{CFT}_4$ where the $\mathcal{N}=1$ formulation of $\mathcal{N}=4$ SYM has been an extremely efficient tool for studies of anomalous dimensions, non-renormalization properties and integrability. We believe that our $\mathcal{N}=1$ action will be similarly useful in this case too.

As a simple test for our action we follow [7] and demonstrate that giving an expectation value in one of the scalar superfields, our action yields the maximally supersymmetric YM theory in $2+1$ dimensions, as it should.

2. The $\mathcal{N}=1$ superfield action

We consider eight real $\mathcal{N}=1$ superfields[2] as

$$\Phi_a^I=\phi_a^I+\theta^{\alpha,\dot 8}\hat\Gamma^I_{\dot 8 A}\psi^A_{\alpha,a}-\theta^2F_a^I,\quad I,A=1,2,\ldots,8, \tag{1}$$

where a denotes the index of the three-algebra algebra with structure constants f^{abcd}. We use the $SO(8)$ triality tensor $\Gamma^I_{A\dot A}$, $I,\dot A,A=1,2,\ldots,8$. In the representation where $\hat\Gamma^I_{\dot 8 A}=-\delta^I_A$ (see Appendix B), choosing the superspace coordinate to point in the $\dot 8$ direction we have essentially made equivalent the vector and one of the two spinorial representations of $SO(8)$. We use the notation of [46] summarized along with other useful relations and conventions in Appendix A.

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[1] For a general discussion see [1].

[2] Earlier works on the superfield formulation of Chern–Simons theories coupled to matter include [44,45].

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Our superfield action couples the matter superfields to a Chern–Simons gauge superfield Γ^α_{ab} in the Wess–Zumino gauge, that carries two gauge group indices a, b

$$S = \int d^3x\, d^2\theta \big[2\gamma\big(D^\alpha \Phi^I_d - f^{abc}{}_d \Gamma^\alpha{}_{ab}\Phi^I_c\big)^2 + \alpha f^{abcd}\big(D^\alpha \Gamma^\beta{}_{ab}\big)(D_\beta \Gamma_{\alpha cd}) + \beta f^{cda}{}_g f^{efgb}\big(D^\alpha \Gamma^\beta{}_{ab}\big)\Gamma_{\alpha cd}\Gamma_{\beta ef} + k f^{abcd} C_{IJKL}\Phi^I_a \Phi^J_b \Phi^K_c \Phi^L_d\big]. \tag{2}$$

Apart from the overall normalization, the only adjustable parameter is the coupling constant of the real superpotential. Nevertheless, we keep all coefficients α, β, γ, k arbitrary having in mind possible generalizations of the action (2).

The crucial point is the use of the self-dual[3] eight-dimensional tensor C_{IJKL}, $I, J, K, L = 1, 2, \ldots, 8$ that describes the embeddings of $SO(7)$ into $SO(8)$ [47–50]. Its properties are briefly recalled in Appendix B. Hence, the presence of the superpotential implies that (2) has only $SO(7)$ manifest global symmetry. Our strategy is to fix the coefficients in (2) by comparing the resulting component action with the one of BL. In this way, the $SO(7)$ symmetry is enhanced to $SO(8)$ and hence we achieve maximal supersymmetry.

We detail next the various projections.

(A) *Scalar kinetic terms*

$$\begin{aligned}
&D^2\big[2\gamma\big(D^\alpha \Phi^I_d - f^{abc}{}_d \Gamma^\alpha{}_{ab}\Phi^I_c\big)^2\big]\big|_{\theta=0} \\
&\quad = -2\gamma F^I{}_d F^{Id} - 2\gamma \phi^I{}_d \Box \phi^{Id} - 2\gamma i \psi^{\alpha I}{}_d \partial_\alpha{}^\beta \psi_\beta{}^{Id} + 2\gamma f^{abc}{}_d f^{efgd} A^\mu{}_{ab} A_{\mu ef}\phi^I{}_c \phi^I{}_g \\
&\qquad - \gamma f^{abc}{}_d\big[-\phi^I{}_c(\gamma_\mu)^\alpha{}_\gamma A^\mu{}_{ab}\partial^\gamma{}_\alpha \phi^{Id} - \big(\partial^\alpha{}_\gamma \phi^{Id}\big)(\gamma_\mu)^\gamma{}_\alpha A^\mu{}_{ab}\phi^I{}_c + 2i\psi^\gamma{}_c{}^I(\gamma_\mu)_\gamma{}^\alpha A^\mu{}_{ab}\psi^{Id}_\alpha + 4\lambda^\alpha{}_{ab}\psi^{Id}_\alpha \phi^I_c\big] \\
&\quad = 2\gamma\big(\nabla^\mu \phi^I{}_d\big)\big(\nabla_\mu \phi^{Id}\big) - 2i\gamma \psi^\alpha{}_d{}^I \nabla^\beta_\alpha \psi^{Id}_\beta - 2\gamma F^I{}_d F^{Id} - 4\gamma f^{abc}{}_d \lambda^\alpha{}_{ab}\psi^{Id}_\alpha \phi^I_c,
\end{aligned} \tag{3}$$

where

$$\nabla_\mu \phi^I_d \equiv \partial_\mu \phi^{Id} - f^{abc}{}_d A_{\mu ab}\phi^{Ic}. \tag{4}$$

(B) *Chern–Simons terms*:

$$\begin{aligned}
&D^2\big[\alpha f^{abcd}\big(D^\alpha \Gamma^\beta{}_{ab}\big)(D_\beta \Gamma_{\alpha cd}) + \beta f^{cda}{}_g f^{efgb}\big(D^\alpha \Gamma^\beta{}_{ab}\big)\Gamma_{\alpha cd}\Gamma_{\beta ef}\big]\big|_{\theta=0} \\
&\quad = 4\alpha f^{abcd}\lambda^\alpha{}_{ab}\lambda_{\alpha cd} - 4\alpha f^{abcd}\epsilon^{\mu\nu\rho}A_{\mu ab}\partial_\nu A_{\rho cd} - 2\beta f^{cda}{}_g f^{efgb}\epsilon^{\mu\nu\rho}A_{\mu ab}A_{\nu cd}A_{\rho ef}.
\end{aligned} \tag{5}$$

(C) *Superpotential*:

$$D^2\big[k f^{abcd} C_{IJKL}\Phi^I{}_a \Phi^J{}_b \Phi^K{}_c \Phi^L{}_d\big]\big|_{\theta=0} = k f^{abcd} C_{IJKL}\big[6\psi^\alpha{}_a{}^I \psi^J_{\alpha b}\phi^K{}_c \phi^L{}_d + 4F^I{}_a \phi^J{}_b \phi^K{}_c \phi^L{}_d\big]. \tag{6}$$

The equations of motion for the auxiliary fields read

$$F^{Ia} = \frac{k}{\gamma} f^{abcd} C^I{}_{JKL}\phi^J{}_b \phi^K{}_c \phi^L{}_d, \tag{7}$$

$$\lambda_{\alpha ab} = \frac{\gamma}{2\alpha}\phi^I{}_a \psi^I{}_{\alpha b}. \tag{8}$$

Using (7) and (8), the terms contributing to the potential from (A), (B) and (C) that involve the auxiliary fields give

$$\begin{aligned}
&-\frac{\gamma^2}{\alpha} f^{abcd}\psi^{\alpha I}_a \phi^I{}_b \psi^J_{\alpha c}\phi^J{}_d + \frac{2k^2}{\gamma} f^{bcda} f^{efg}{}_a C^I{}_{OMN} C_{IJKL}\phi^O{}_e \phi^M{}_f \phi^N{}_g \phi^J{}_b \phi^K{}_c \phi^L{}_d \\
&\quad = -\frac{\gamma^2}{\alpha} f^{abcd}\psi^{\alpha I}_a \phi^I{}_b \psi^J_{\alpha c}\phi^J{}_d + \frac{12k^2}{\gamma} f^{bcda} f^{efg}{}_a \phi^J{}_e \phi^K{}_f \phi^L{}_g \phi^J{}_b \phi^K{}_c \phi^L{}_d - \frac{18k^2}{\gamma} f^{bcda} f^{efg}{}_a C_{KLOM}\phi^J{}_e \phi^K_f \phi^L_g \phi^O{}_b \phi^M{}_c \phi^J{}_d \\
&\quad = -\frac{\gamma^2}{\alpha} f^{abcd}\psi^{\alpha I}_a \phi^I{}_b \psi^J_{\alpha c}\phi^J{}_d + \frac{12k^2}{\gamma}\mathrm{Tr}\big(\big[\phi^J, \phi^K, \phi^L\big], \big[\phi^J, \phi^K, \phi^L\big]\big).
\end{aligned} \tag{9}$$

To arrive at the last line we have used the contraction of the self-dual form [47,48,50]

$$C^I{}_{OMN} C^{IJKL} = \delta^{[J}{}_O \delta^K{}_M \delta^{L]}{}_N - 9C^{KL}{}_{OM}\delta^J_N. \tag{10}$$

It is also crucial that the third line in (9) vanishes. This can be shown for a general 3-algebra with totally antisymmetric structure constants f^{abcd}. Indeed, using the fundamental identity

$$f^{bcda} f^{efg}{}_a = f^{efda} f^{bcg}{}_a + f^{efba} f^{cdg}{}_a + f^{efca} f^{dbg}{}_a \tag{11}$$

in the relevant part of the third line of (9) we obtain after some relabeling

$$f^{bcda} f^{efg}{}_a C_{KLOM}\phi^J{}_e \phi^K_f \phi^L_g \phi^O{}_b \phi^M{}_c \phi^J{}_d = -2 f^{bcga} f^{efd}{}_a C_{KLOM}\phi^J_e \phi^K_f \phi^L_g \phi^O_b \phi^M_d \phi^J_c = -2 f^{bcda} f^{efg}{}_a C_{KLOM}\phi^J_e \phi^K_f \phi^L_g \phi^O_b \phi^M_c \phi^J_d, \tag{12}$$

where we have also used the antisymmetry of C_{KLOM}. Hence this term vanishes.

Putting everything together we arrive at the component action

[3] Similar result can be obtained using the anti-self-dual tensor.

$$S=\int d^3x\Big[2\gamma\big(\nabla^\mu\phi^I{}_d\big)\big(\nabla_\mu\phi^{Id}\big)-2\gamma i\psi^{\alpha I}{}_d\nabla_\alpha^\beta\psi_\beta^{Id}-4\alpha f^{abcd}\epsilon^{\mu\nu\rho}A_{\mu ab}\partial_\nu A_{\rho cd}-2\beta f^{cda}{}_g f^{efgb}\epsilon^{\mu\nu\rho}A_{\mu ab}A_{\nu cd}A_{\rho ef}$$
$$+6kf^{abcd}\bigg(C_{IJKL}+\frac{\gamma^2}{6k\alpha}\delta_{IK}\delta_{JL}\bigg)\psi^\alpha{}_a{}^I\psi^J_{\alpha b}\phi^K{}_c\phi^L{}_d+\frac{12k^2}{\gamma}\mathrm{Tr}\big(\big[\phi^I,\phi^J,\phi^K\big],\big[\phi^I,\phi^J,\phi^K\big]\big)\bigg]. \tag{13}$$

Now, in our representation we have

$$\psi_a^{\alpha I}=\hat\Gamma^I_{\dot 8A}\psi_a^{\alpha A}=-\delta^I_A\psi_a^{\alpha A}, \tag{14}$$

so that for the spinor kinetic term we have

$$-2\gamma i\psi^{\alpha I}{}_d\nabla_\alpha^\beta\psi_\beta^{Id}=-2\gamma i\psi^{\alpha A}{}_d\nabla_\alpha^\beta\psi_\beta^{Ad}. \tag{15}$$

Then, choosing

$$\alpha=-\frac{1}{8},\qquad \beta=-\frac{1}{6},\qquad \gamma=-\frac{1}{4},\qquad k=-\frac{1}{24}, \tag{16}$$

we obtain the action

$$S=\int d^3x\bigg[-\frac{1}{2}\big(\nabla^\mu\phi^I{}_d\big)\big(\nabla_\mu\phi^{Id}\big)+\frac{i}{2}\psi^{\alpha A}{}_d\nabla_\alpha^\beta\psi_\beta^{Ad}+\frac{1}{2}f^{abcd}\epsilon^{\mu\nu\rho}A_{\mu ab}\partial_\nu A_{\rho cd}+\frac{1}{3}f^{cda}{}_g f^{efgb}\epsilon^{\mu\nu\rho}A_{\mu ab}A_{\nu cd}A_{\rho ef}$$
$$-\frac{1}{4}f^{abcd}(C_{IJKL}+\delta_{IK}\delta_{JL}-\delta_{IL}\delta_{JK})\psi^\alpha{}_a{}^I\psi^J_{\alpha b}\phi^K{}_c\phi^L{}_d-\frac{1}{12}\mathrm{Tr}\big(\big[\phi^I,\phi^J,\phi^K\big],\big[\phi^I,\phi^J,\phi^K\big]\big)\bigg]. \tag{17}$$

This coincides exactly with the Bagger and Lambert action given in [3]. To see that, notice that in our notations we use the purely imaginary charge conjugation matrix C to raise and lower spinor indices. Therefore for a real Majorana spinor ψ we have the identifications

$$(\bar\psi)^\alpha=\big(\psi^TC\big)^\alpha=C^{\alpha\beta}\psi_\beta=\psi^\alpha=i(\bar\psi_{\mathrm{BL}})^\alpha, \tag{18}$$

and also to match the fermion kinetic term

$$i\psi^\alpha\big(\gamma^\mu\partial_\mu\big)_\alpha{}^\beta\psi_\beta=(\bar\psi_{\mathrm{BL}})^\alpha\big(\gamma^\mu\partial_\mu\big)_\alpha{}^\beta(\psi_{\mathrm{BL}})_\beta=i(\bar\psi_{\mathrm{BL}})^\alpha\big(\gamma^\mu_{\mathrm{BL}}\partial_\mu\big)_\alpha{}^\beta(\psi_{\mathrm{BL}})_\beta, \tag{19}$$

that requires (note the position of the γ-matrix indices)

$$\big(\gamma^\mu_{\mathrm{BL}}\big)_\alpha{}^\beta=i\big(\gamma^\mu\big)^\beta_\alpha. \tag{20}$$

Our final superspace action is then written as

$$S=\int d^3x\,d^2\theta\bigg[-\frac{1}{2}\big(D^\alpha\Phi^I_d-f^{abc}{}_d\Gamma^\alpha{}_{ab}\Phi^I_c\big)^2-\frac{1}{8}f^{abcd}\big(D^\alpha\Gamma^\beta{}_{ab}\big)(D_\beta\Gamma_{\alpha cd})$$
$$-\frac{1}{6}f^{cda}{}_g f^{efgb}\big(D^\alpha\Gamma^\beta{}_{ab}\big)\Gamma_{\alpha cd}\Gamma_{\beta ef}-\frac{1}{24}f^{abcd}C_{IJKL}\Phi^I_a\Phi^J_b\Phi^K_c\Phi^L_d\bigg]. \tag{21}$$

3. M2 to D2

As a simple test for our action (21) we give an expectation value to one of the scalars. Following [7] we expect that the resulting action will be the maximally supersymmetric YM theory in $2+1$ dimensions for the gauge group $SU(2)$. We split the $I=1,\dots,8$ index of the scalar superfield as $I\mapsto(i,8)$ with $i=1,\dots,7$ and the $SO(4)$ index $\hat a=1,\dots,4$ as $\hat a\mapsto(a,x)$ with $a=1,\dots,3$. Then we give an expectation value to the scalar superfield which we identify with the dimensionful coupling constant of the $2+1$ SYM as

$$\big\langle\Phi^8_x\big\rangle=g_{\mathrm{YM}}. \tag{22}$$

For the spinor superfield we define

$$\Gamma^\alpha_{ax}\equiv A^\alpha_a, \tag{23}$$
$$\epsilon^{abc}\Gamma^\alpha_{ab}\equiv B^\alpha_c. \tag{24}$$

We rewrite the superspace action in terms of the new fields and indices.
(i) *Kinetic terms*:

$$2\gamma\big(D^\alpha\Phi^I_{\hat d}-\epsilon^{\hat a\hat b\hat c}{}_{\hat d}\Gamma^\alpha{}_{\hat a\hat b}\Phi^I_{\hat c}\big)^2=\gamma\big[\nabla^\alpha\Phi^i_d\nabla_\alpha\Phi^{id}+\nabla^\alpha\Phi^8_d\nabla_\alpha\Phi^{8d}+D^\alpha\Phi^i_xD_\alpha\Phi^{ix}$$
$$+B^\alpha_d\big(2\Phi^i_x\nabla_\alpha\Phi^{id}+2g_{\mathrm{YM}}\nabla_\alpha\Phi^{8d}-2\Phi^{id}D_\alpha\Phi^{ix}\big)+B^\alpha_dB^d_\alpha\big(g^2_{\mathrm{YM}}+\Phi^i_x\Phi^i_x\big)+B^{\alpha c}B^g_\alpha\Phi^i_c\Phi^i_g\big], \tag{25}$$

where we defined the gauge covariant derivative

$$\nabla^\alpha\Phi^i_d=D^\alpha\Phi^i_d-2\epsilon^{bcd}A^\alpha_b\phi^i_c. \tag{26}$$

(ii) *Superpotential term*:

$$k\epsilon^{\hat a\hat b\hat c\hat d}C_{IJKL}\Phi^I_{\hat a}\Phi^J_{\hat b}\Phi^K_{\hat c}\Phi^L_{\hat d}=4k\epsilon^{bcd}c_{jkl}g_{\mathrm{YM}}\Phi^j_b\Phi^k_c\Phi^l_d+\cdots, \tag{27}$$

where the dots indicate subleading terms in the large g_{YM} limit which we have discarded.

(iii) *Chern–Simons terms*:

$$\alpha\epsilon^{\hat a\hat b\hat c\hat d}\big(D^\alpha\Gamma^\beta{}_{\hat a\hat b}\big)D_\beta\Gamma_{\alpha\hat c\hat d}+\beta\epsilon^{\hat c\hat d\hat a}{}_{\hat g}\epsilon^{\hat e\hat f\hat g\hat b}\big(D^\alpha\Gamma^\beta{}_{\hat a\hat b}\big)\Gamma_{\alpha\hat c\hat d}\Gamma_{\beta\hat e\hat f}$$
$$=4\alpha B^{\beta d}\big(D^\alpha D_\beta A_{\alpha d}\big)+2\beta\epsilon^{dag}B^\beta_g\big[2\big(D^\alpha A_{\beta a}\big)A_{\alpha d}+\big(D_\beta A^\alpha_a\big)A_{\alpha d}+\big(D^\alpha A_{\alpha a}\big)A_{\beta d}\big]+\frac{\beta}{2}\epsilon^{abc}\big(D^\alpha B^\beta_c\big)B_{\alpha a}B_{\beta b}$$
$$=B^\beta_d\big[4\alpha D^\alpha D_\beta A^d_\alpha+6\beta\epsilon^{dba}\big(D^\alpha A_{\beta a}\big)A_{\alpha b}\big]+\frac{\beta}{2}\epsilon^{abc}\big(D^\alpha B^\beta_c\big)B_{\alpha a}B_{\beta b}. \tag{28}$$

To arrive in the last line we have used the fact that in the Wess–Zumino gauge $D^\alpha\Gamma_\alpha$ is vanishing and $D_\alpha\Gamma_\beta$ can be symmetrized.

Next, we derive the equations of motions for the B auxiliary superfield neglecting the terms cubic in B and also terms of the form $B^2\Phi^2$, as in [7]. We thus obtain

$$B_{\alpha d}=-\frac{1}{2\gamma g^2_{\mathrm{YM}}}\big[W_{\alpha d}+2\gamma\big(\Phi^i_x\nabla_\alpha\Phi^i_d+g_{\mathrm{YM}}\nabla_\alpha\Phi^8_d-\Phi^i_d D_\alpha\Phi^{ix}\big)\big], \tag{29}$$

where we defined

$$W^d_\beta=4\alpha D^\alpha D_\beta A^d_\alpha+6\beta\epsilon^{dba}\big(D^\alpha A_{\beta a}\big)A_{\alpha b}. \tag{30}$$

Inserting B into the action and collecting all the terms we get

$$\mathcal{L}=-\frac{1}{2\gamma g^2_{\mathrm{YM}}}\big[W^\alpha_d+2\gamma\big(\Phi^i_x\nabla^\alpha\Phi^i_d+g_{\mathrm{YM}}\nabla^\alpha\Phi^8_d-\Phi^i_d D^\alpha\Phi^{ix}\big)\big]^2+4k\epsilon^{bcd}c_{jkl}g_{\mathrm{YM}}\Phi^j_b\Phi^k_c\Phi^l_d$$
$$+\gamma\big[\nabla^\alpha\Phi^i_d\nabla_\alpha\Phi^{id}+\nabla^\alpha\Phi^8_d\nabla_\alpha\Phi^{8d}+D^\alpha\Phi^i_x D_\alpha\Phi^{ix}\big]$$
$$=\frac{1}{g^2_{\mathrm{YM}}}W^\alpha_d W^d_\alpha-\frac14\nabla^\alpha\Phi^i_d\nabla_\alpha\Phi^{id}-\frac14 D^\alpha\Phi^i_x D_\alpha\Phi^{ix}-\frac16\epsilon^{bcd}c_{jkl}g_{\mathrm{YM}}\Phi^j_b\Phi^k_c\Phi^l_d+\dots, \tag{31}$$

where the dots stand for subleading contributions and we substituted the values in (16), so that now

$$W^d_\beta=\frac12 D^\alpha D_\beta A^d_\alpha+\epsilon^{dba}\big(D^\alpha A_{\beta a}\big)A_{\alpha b}. \tag{32}$$

Rescaling the gauge superfield as in [7] $A\to 1/2A$ we see that $W^\alpha_d W^d_\alpha$ gives the right SYM kinetic term in the Wess–Zumino gauge and the covariant derivative in (26) assumes its standard form. Finally, ignoring the contribution of Φ^i_x that has completely decoupled, we arrive at

$$\mathcal{L}=\frac{1}{8g^2_{\mathrm{YM}}}\big(D^\alpha D_\beta A^d_\alpha+\epsilon^{dba}\big(D^\alpha A_{\beta a}\big)A_{\alpha b}\big)^2-\frac14\nabla^\alpha\Phi^i_d\nabla_\alpha\Phi^{id}-\frac16\epsilon^{bcd}c_{jkl}g_{\mathrm{YM}}\Phi^j_b\Phi^k_c\Phi^l_d. \tag{33}$$

This is the superfield Lagrangian for maximally supersymmetric YM in $2+1$ dimensions for $SU(2)$. It is intriguing to notice that the Lagrangian (33) is remarkably similar to the octonionic $\mathcal{N}=(1,1)$ sigma model (in two dimensions) of [51].

4. Conclusions

We have presented an $\mathcal{N}=1$ superfield action in three dimensions that in components gives the Bagger–Lambert action for a general 3-algebra with totally antisymmetric structure constants f^{abcd}. Crucial in our construction were the self-dual octonionic tensors C_{IJKL}. Although the tensors are $SO(7)$ invariant, we have shown that a special choice of the parameters in the action enhances the global symmetry to $SO(8)$. We have demonstrated that a superhiggs mechanism yields the maximally supersymemtric $(2+1)$ YM theory on D2 branes, curiously in a formalism resembling a two-dimensional sigma model. We hope that our superfield action and its generalizations can be used in $\mathcal{N}=1$ superfield calculations that should shed more light into the $\mathrm{AdS}_4/\mathrm{CFT}_3$ correspondence.

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Appendix A. Superspace notations

In this appendix we collect the useful identities for our superfields and gamma matrices. We follow Superspace [46].

The component field definitions are as

$$\begin{aligned}
&\Phi^I_a\big|\equiv\phi^I_a, && D_\alpha\Phi^I_a\big|\equiv\psi^I_{\alpha a},\\
&D^2\Phi^I_a\big|\equiv F^I_a, && \Gamma_{\alpha ab}\big|\equiv\chi_{\alpha ab},\\
&\frac12 D^\alpha\Gamma_{\alpha ab}\big|\equiv B_{ab}, && D^2\Gamma_{\alpha ab}\big|\equiv 2\lambda_{\alpha ab}-i\partial^\beta_\alpha\chi_{\beta ab},\\
&D_\alpha\Gamma^\beta_{ab}\big|\equiv i(\gamma_\mu)^\beta_\alpha A^\mu{}_{ab}-\delta^\beta_\alpha B_{ab}, && D^2\Gamma^\alpha_{ab}\big|\equiv 2\lambda^\alpha_{ab}+i\partial^\alpha_\beta\chi^\beta_{ab},\\
&D^\alpha\Gamma_{\beta ab}\big|\equiv i(\gamma_\mu)^\alpha_\beta A^\mu_{ab}+\delta^\alpha_\beta B_{ab}, && \frac12 D^\beta D_\alpha\Gamma_{\beta ab}\big|\equiv\lambda_{\alpha ab}.
\end{aligned}$$

In the analog of the Wess–Zumino procedure we can gauge away B and χ component fields. Our spacetime signature is $(-,+,+)$. The purely immaginary totally antisymmetric symbol $C_{\alpha\beta}$ is used to raise and lower spinor indexes according to the $\searrow$ convention

$$C_{\alpha\beta}=-C_{\beta\alpha}=-C^{\alpha\beta}=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix},\qquad C_{\alpha\beta}C^{\gamma\delta}=\delta^{\gamma}_{[\alpha}\delta^{\delta}_{\beta]},$$

$$\psi_\alpha=\psi^\beta C_{\beta\alpha},\qquad \psi^\alpha=C^{\alpha\beta}\psi_\beta,\qquad \psi^2=\frac{1}{2}\psi^\alpha\psi_\alpha.$$

We represent vectors in spinor notation as symmetric matrices:

$$(\gamma^0)_{\alpha\beta}=-(\gamma^0)^{\alpha\beta}=-(\gamma_0)_{\alpha\beta}=(\gamma_0)^{\alpha\beta}=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix},$$

$$(\gamma^1)_{\alpha\beta}=(\gamma^1)^{\alpha\beta}=(\gamma_1)_{\alpha\beta}=(\gamma_1)^{\alpha\beta}=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix},$$

$$(\gamma^2)_{\alpha\beta}=(\gamma^2)^{\alpha\beta}=(\gamma_2)_{\alpha\beta}=(\gamma_2)^{\alpha\beta}=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix},$$

$$(\gamma^\mu)_{\alpha\beta}V_\mu=\begin{pmatrix}V_0+V_1 & V_2\\ V_2 & V_0-V_1\end{pmatrix}.$$

Then the following relations hold:

$$\partial^{\alpha\beta}=(\gamma_\mu)^{\alpha\beta}\partial^\mu,\qquad \partial^\mu=\frac{1}{2}(\gamma^\mu)_{\alpha\beta}\partial^{\alpha\beta},$$

$$x^{\alpha\beta}=\frac{1}{2}(\gamma_\mu)^{\alpha\beta}x^\mu,\qquad x^\mu=(\gamma^\mu)_{\alpha\beta}x^{\alpha\beta},$$

$$A^{\alpha\beta}=\frac{1}{\sqrt{2}}(\gamma_\mu)^{\alpha\beta}A^\mu,\qquad A^\mu=\frac{1}{\sqrt{2}}(\gamma^\mu)_{\alpha\beta}A^{\alpha\beta},$$

$$\partial_{\alpha\beta}x^{\gamma\rho}=\frac{1}{2}\delta^{\gamma}_{(\alpha}\delta^{\rho}_{\beta)},\qquad \partial_\alpha\theta^\beta=\delta_\alpha{}^\beta,$$

$$(\gamma_\mu)^{\alpha\beta}(\gamma_\nu)_{\alpha\beta}=2\eta_{\mu\nu},\qquad (\gamma^\mu)_{\alpha\beta}(\gamma_\mu)^{\gamma\rho}=\delta^\gamma_\alpha\delta_\beta{}^\rho+\delta_\alpha{}^\rho\delta_\beta{}^\gamma,$$

$$(\gamma^\mu)^{\alpha\beta}(\gamma^\nu)_{\beta\rho}=\eta^{\mu\nu}\delta^\alpha_\rho+i\epsilon^{\mu\nu\sigma}(\gamma_\sigma)^\alpha{}_\rho,\qquad (\gamma^\mu)^\rho{}_\rho=0,$$

$$(\gamma^\mu)^{\alpha\beta}(\gamma^\nu)_\beta{}^\rho(\gamma^\sigma)_{\rho\alpha}=2i\epsilon^{\mu\nu\sigma},\qquad \epsilon^{012}=1.$$

We conclude with some useful relations for three-dimensional D-algebra computations:

$$D^\alpha D_\beta=i\partial^\alpha{}_\beta+\delta^\alpha{}_\beta D^2,\qquad D_\alpha D^\beta=i\partial_\alpha{}^\beta-\delta^\beta_\alpha D^2,$$

$$D^2D_\alpha=-i\partial_\alpha{}^\beta D_\beta,\qquad D_\alpha D^2=i\partial_\alpha{}^\beta D_\beta,$$

$$D^2D^\alpha=i\partial^\alpha{}_\beta D^\beta,\qquad D^\alpha D^2=-i\partial^\alpha{}_\beta D^\beta,$$

$$D^\alpha D_\beta D_\alpha=0,\qquad \partial^{\alpha\beta}\partial_{\gamma\beta}=\delta^\alpha{}_\gamma\Box,$$

$$D^2D^2=\Box,\qquad D^\alpha D^2 D_\alpha=-2\Box,$$

$$D^2D_\alpha D^2=-\Box D_\alpha,\qquad \Box=\partial^\mu\partial_\mu=\frac{1}{2}\partial^{\alpha\beta}\partial_{\alpha\beta}.$$

Appendix B. Octonionic conventions

In the octonion algebra we can choose a basis of elements

$$\{1,e_i\},\quad i=1,\dots,7$$

such that

$$e_ie_j=c_{ijk}e_k-\delta_{ij},$$

where the tensor c_{ijk} is totally antisymmetric with non-vanishing entries:

$$c_{123}=c_{147}=c_{165}=c_{246}=c_{257}=c_{354}=c_{367}=1.$$

We can also introduce the seven-dimensional dual of the structure constants:

$$c_{ijkl}=\frac{1}{6}\epsilon_{ijklmno}c^{mno}.$$

Combining these two objects one can construct an $SO(7)$ invariant tensor $C_{IJKL\ I,J,K,L=1,\dots,8}$ which is self dual in 8 dimensions, by taking:

$$C_{ijk8}=c_{ijk},\qquad C_{ijkl}=c_{ijkl}.$$

Octonionic structure constants can be used to construct $SO(8)$ gamma matrices. For instance a suitable representation of the triality tensor that enters BL susy transformations is given by:

$$(\Gamma^i)_{A\dot{A}} = c^i{}_{A\dot{A}} + \delta_{8\dot{A}}\delta_{Ai} - \delta_{8A}\delta_{\dot{A}i}, \quad i = 1, \dots, 7, \quad A, \dot{A} = 1, \dots, 8,$$
$$(\Gamma^8)_{A\dot{A}} = \delta_{A\dot{A}}, \qquad c^i_{8\dot{A}} = c^i_{A8} = 0.$$

Defining $\hat{\Gamma}^I_{\dot{A}A} = (\Gamma^T)^I_{\dot{A}A}$ it is easy to see that $\Gamma^I \hat{\Gamma}^J + \Gamma^J \hat{\Gamma}^I = 2\delta^{IJ}$ and therefore we can write down 16×16 matrices satisfying the Clifford algebra

$$\gamma^I = \begin{pmatrix} 0 & \Gamma^I{}_{A\dot{A}} \\ \hat{\Gamma}^I{}_{\dot{A}A} & 0 \end{pmatrix}, \qquad \gamma^I\gamma^J + \gamma^J\gamma^I = 2\delta^{IJ}.$$

With the above definitions it can be shown that

$$\Gamma^{IJ}{}_{AB} = \frac{1}{2}\left(\Gamma^I_{A\dot{A}}\hat{\Gamma}^J_{\dot{A}B} - \Gamma^J_{A\dot{A}}\hat{\Gamma}^I_{\dot{A}B}\right) = C^{IJ}{}_{AB} + \delta^I_A\delta^J_B - \delta^I_B\delta^J_A,$$

so that the antisymmetrized product of Γ's of the scalar-fermion interaction can be written by means of the C_{IJKL} tensor.

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